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## Radiative Corrections to the Higgs Boson Decay into a Longitudinal $W$ -Boson Pair in a Two-Doublet Model

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### Abstract

The decay rate of a (neutral) Higgs boson ( $H$ ) going into a longitudinal  $W$ -boson pair is calculated by including one loop radiative corrections in a two-Higgs doublet model. It is assumed that the Higgs boson  $H$  is much heavier than the  $W$  boson and a full use has been made of the equivalence theorem. A possibility is explored extensively that the precise measurement of the decay rate of  $H$  could be useful as a probe of the other neutral as well as charged Higgs bosons through potentially important radiative corrections. It is pointed out that, for some choice of the parameters, the radiative corrections are sizable and that there exist a possibility of extracting useful information on the other Higgs boson masses.

## 1. Introduction

In the last two decades there have been a lot of efforts towards discovery of Higgs bosons both theoretically and experimentally. (See Ref. [1] for a review.) There is potentially a hope on the theoretical side that, even before direct detections of the Higgs bosons, one might be able to get indirect signatures of the existence of the Higgs particles: *i.e.*, the study of radiative corrections and precision tests of electro-weak processes have been often motivated to see low-energy manifestations of unknown very heavy particles.

Veltman [2] has once argued that in the minimal standard model with a single Higgs boson doublet, the Higgs boson mass dependence of radiative corrections would be at most logarithmic on the one-loop level and therefore the internal loop effects of the Higgs boson are rather elusive. Einhorn and Wudka [3] studied radiative corrections to gauge boson propagators in a general way and strengthened the Veltman's claim. They realized that, due to the custodial  $SU(2)$  symmetry in a single Higgs scheme [4], the Higgs mass dependence is reduced to a logarithmic one to all orders in perturbation theory. This fact is sometimes referred to as Veltman's screening theorem.

The analysis of Einhorn and Wudka tells us that radiative corrections other than those to vector-boson propagators may have power-type terms w.r.t. Higgs masses. It is also clear that, in a model where the custodial symmetry is not respected, there is no compelling reason a priori that Higgs bosons do not produce power-like corrections to gauge boson propagators. Two-Higgs doublet models belong to such examples and the calculation performed by Toussaint [5] shows in fact that the screening theorem does not apply to the two-doublet model. This fact prompted several authors [6-7] to study radiative corrections to the  $\rho$ -parameter and to the muon decay constant in a two-doublet model. Analyses along the line of Peskin and Takeuchi [8] are also available [9-10]. All of these analyses are, however, restricted

to the oblique-type corrections. It is now apparent that other processes should be investigated in the two-doublet model as much as possible from the view point of the internal loop effects of Higgs boson masses. The two-doublet model could allow for large radiative corrections and would provide us with useful information.

On the experimental side, it has been considered that longitudinally polarized gauge boson scatterings, i.e.,  $W_L^+W_L^- \rightarrow W_L^+W_L^-$ ,  $W_L^+W_L^- \rightarrow Z_LZ_L$ , are suitable processes to discover Higgs bosons in future hadron colliders [11]. An elaborate calculations of these reactions have been carried out in the minimal standard model (with a single Higgs doublet) including higher order radiative corrections [12-14]. The decay process  $H(\text{Higgs}) \rightarrow W_L^+W_L^-$  constitutes sub-diagrams of the gauge boson scatterings and the decay width has also been evaluated in the minimal standard model. (For experimental aspects of measuring the decay width, see Ref. [15].)

Considering the importance of the  $WW$ -scattering and the decay process, we have decided in the present paper to undertake the calculation of the Higgs decay width into a  $W_L^+W_L^-$  pair in the two-Higgs doublet model including loop corrections. Our theoretical motivation is to scrutinize the dependence of the width on various Higgs boson masses and explore the possibility of getting indirect signature of Higgs bosons other than  $H$ , which are yet to be discovered. There are two CP-even neutral Higgs bosons ( $H, h$ ) and a CP-odd one ( $A$ ) together with a charged one ( $G^\pm$ ) in the two Higgs doublet model. In the present paper, we will assume that the  $H$ -boson is the lightest among these scalar particles but is still much heavier than the  $W$ -boson. We will study the internal loop effects due to the other heavier scalar particles,  $h$ ,  $A$ , and  $G^\pm$  to the decay process  $H \rightarrow W_L^+W_L^-$ .

There have been several attempts to get information on the masses of these bosons. According to the tree unitarity analysis [16] of the type of Lee, Quigg and Thacker [17], these masses are bounded from above as  $m_H < 500\text{GeV}$ ,  $m_h < 710\text{GeV}$ ,  $m_G < 870\text{GeV}$ ,  $m_A < 1200\text{GeV}$ . This is a criterion of the validity of

perturbation theory. Similar bounds are also obtained on the basis of triviality arguments [18]. Suppose that the lightest Higgs boson  $H$  has been discovered in a future collider. The reasonable question to be raised thereby will be whether one could glimpse into the existence of another Higgs boson in the mass range mentioned above by looking at the decay width of the discovered  $H$ . In such a situation the calculation in the present paper will become very useful.

This paper is organized as follows. We describe the two-Higgs model in Sec. 2 and discuss the outline of our calculation. We then proceed in Sec. 3 to calculate the tadpole diagrams and various two-point functions. In Sec. 4 we give the calculation of the Higgs decay vertex function. Numerical analyses of the decay width formula are given in Sec. 5. Sec. 6 is devoted to summary and discussions.

## 2. The Two-Higgs Doublet Model

Let us begin with the Higgs potential, consisting of two Higgs doublets,  $\Phi_1$  and  $\Phi_2$  with  $Y = 1$ . The most general  $SU(2)_L \times U(1)_Y$  invariant Higgs potential becomes [19]

$$\begin{aligned}
 V(\Phi_1, \Phi_2) = & -\mu_1^2 |\Phi_1|^2 - \mu_2^2 |\Phi_2|^2 - (\mu_{12}^2 \Phi_1^\dagger \Phi_2 + \mu_{12}^{2*} \Phi_2^\dagger \Phi_1) \\
 & + \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 \\
 & + \lambda_4 (\text{Re} \Phi_1^\dagger \Phi_2)^2 + \lambda_5 (\text{Im} \Phi_1^\dagger \Phi_2)^2.
 \end{aligned} \tag{1}$$

It is imperative to avoid the flavor changing neutral current and henceforth the discrete symmetry under  $\Phi_2 \rightarrow -\Phi_2$  [20] has been assumed except for the soft breaking term  $(\mu_{12}^2 \Phi_1^\dagger \Phi_2 + \mu_{12}^{2*} \Phi_2^\dagger \Phi_1)$ . This soft breaking term may be important in connection with a new source of CP-violation for baryon genesis [21]. Inclusion of this term, however, will render our calculation clumsy to some extent, because of the mixing of CP-even and odd states [22]. Only for this reason, we set

$\mu_{12} = \mu_{12}^* = 0$  throughout the present paper. Inclusion of mixing between CP-even and odd states will be discussed in our future publications.

In order to see the particle contents and mass spectrum, we rewrite the Higgs potential in terms of the parametrization of the Higgs doublets

$$\Phi_i = \begin{pmatrix} w_i^\dagger \\ \frac{1}{\sqrt{2}}(v_i + h_i + iz_i) \end{pmatrix}, \quad (2)$$

where the vacuum expectation values  $v_1$  and  $v_2$  triggering the spontaneous break down, are assumed to be positive without spoiling generality.

The mass term in Eq. (1) may be diagonalized by introducing two kinds of mixing angles,  $\alpha$  and  $\beta$  in the following way

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}, \quad (3)$$

$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} w \\ G \end{pmatrix}, \quad (4)$$

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} z \\ A \end{pmatrix}. \quad (5)$$

The mixing angles are determined in fact by the following relations.

$$\tan 2\alpha = \frac{(\lambda_3 + \lambda_4)v_1 v_2}{\lambda_1 v_1^2 - \lambda_2 v_2^2}, \quad \tan \beta = \frac{v_2}{v_1} \quad (6)$$

$(\pi/2 \geq \alpha \geq -\pi/2, \pi/2 > \beta > 0)$ . The Nambu-Goldstone bosons to be absorbed into the longitudinal part of  $W^\pm$  and  $Z$  are denoted by  $w$  and  $z$  respectively. Other fields  $h$ , and  $H$  are neutral while  $G$  is a charged one. The five coupling constants in Eq. (1) are expressed by the masses of these scalar particles together with the mixing angles [23]

$$\lambda_1 = \frac{1}{2v^2 \cos^2 \beta} (m_h^2 \cos^2 \alpha + m_H^2 \sin^2 \alpha), \quad (7)$$

$$\lambda_2 = \frac{1}{2v^2 \sin^2 \beta} (m_h^2 \sin^2 \alpha + m_H^2 \cos^2 \alpha), \quad (8)$$

$$\lambda_3 = \frac{\sin 2\alpha}{v^2 \sin 2\beta} (m_h^2 - m_H^2) + \frac{2m_G^2}{v^2}, \quad (9)$$

$$\lambda_4 = -\frac{2m_G^2}{v^2}, \quad (10)$$

$$\lambda_5 = \frac{2}{v^2} (m_A^2 - m_G^2), \quad (11)$$

where  $v = \sqrt{v_1^2 + v_2^2} \approx 246 \text{ GeV}$ . Incidentally, masses  $\mu_1^2$  and  $\mu_2^2$  in Eq. (1) are given on the tree level by the vacuum stability condition

$$\mu_1^2 = \lambda_1 v_1^2 + \frac{1}{2} (\lambda_3 + \lambda_4) v_2^2, \quad \mu_2^2 = \lambda_2 v_2^2 + \frac{1}{2} (\lambda_3 + \lambda_4) v_1^2. \quad (12)$$

Eqs. (7)-(12) define the change of our initial seven parameters  $(\lambda_i (i = 1 - 5), \mu_1^2, \mu_2^2)$  into the set  $(m_h, m_H, m_G, m_A, \alpha, \beta, v)$ .

We are now interested in the decay process  $H \rightarrow W_L^+ + W_L^-$ . We will use the equivalence theorem [23, 17] which states that if  $m_H^2 \gg M_W^2$ , we are allowed to replace the external longitudinal gauge bosons by the corresponding Nambu-Goldstone bosons. In our case, therefore, the evaluation of the process  $H \rightarrow w^+ w^-$  is our central concern. The tree-level interaction term dictating this process is extracted from the potential (1) as

$$\mathcal{L}_{Hww} = \frac{m_H^2}{v} \sin(\alpha - \beta) H w^\dagger w. \quad (13)$$

One might suspect that the counter terms corresponding to this part of the potential are just obtained by varying the coefficients in (13), that is, by putting  $m_H^2 \rightarrow m_H^2 - \delta m_H^2$ ,  $v \rightarrow v - \delta v$ ,  $\alpha \rightarrow \alpha - \delta\alpha$ , and  $\beta \rightarrow \beta - \delta\beta$ , together with the wave function renormalization. This, however, does not give us all the counter terms. There are other terms coming from the state mixing. Changes of mixing angles  $\delta\alpha$  and  $\delta\beta$  induce a mixture of the paired fields,  $H \leftrightarrow h$ , and  $w \leftrightarrow G$ , respectively. This

indicates that some of the counterterms are obtained from the  $hw^\dagger w$ ,  $HGw^\dagger$ , and  $HG^\dagger w$  vertices

$$\mathcal{L}_{hww} = -\frac{m_h^2}{v} \cos(\alpha - \beta) hw^\dagger w, \quad (14)$$

$$\mathcal{L}_{HGw} = -\frac{m_H^2 - m_G^2}{v} \cos(\alpha - \beta) (Hw^\dagger G + HwG^\dagger), \quad (15)$$

by replacing  $h \rightarrow -\delta\alpha H$  and  $G \rightarrow \delta\beta w$ .

After all we conclude that the counter terms for the  $Hw^\dagger w$  vertex take the following form

$$\begin{aligned} \delta\mathcal{L}_{Hww} = & \left\{ -\frac{\delta m_H^2}{m_H^2} + \frac{\delta v}{v} + (\sqrt{Z_H} - 1) + (Z_w - 1) \right\} \mathcal{L}_{Hww} \\ & + \delta\alpha \frac{m_h^2 - m_H^2}{v} \cos(\alpha - \beta) Hw^\dagger w \\ & + \delta\beta \frac{2m_G^2 - m_H^2}{v} \cos(\alpha - \beta) Hw^\dagger w. \end{aligned} \quad (16)$$

Thus to evaluate the radiative corrections to this process, we have to know  $\delta m_H^2$ ,  $\delta v$ ,  $\delta\alpha$ ,  $\delta\beta$ , together with the wave function renormalization constants  $Z_w$  and  $Z_H$ .

The calculation of  $\delta v/v$  is facilitated by considering the renormalization of the  $W$ -boson mass  $M_W$ , *i.e.*  $\delta v/v = \delta M_W^2/2M_W^2$ . This has been computed previously by Toussaint [5] and we just quote his results,

$$\begin{aligned} \frac{\delta M_W^2}{M_W^2} = & \frac{1}{(4\pi)^2 v^2} \left\{ \frac{1}{2} (2m_G^2 + m_H^2 + m_h^2 + m_A^2) + \frac{m_G^2 m_A^2}{m_A^2 - m_G^2} \ln \frac{m_G^2}{m_A^2} \right. \\ & \left. + \cos^2(\alpha - \beta) \frac{m_G^2 m_H^2}{m_H^2 - m_G^2} \ln \frac{m_G^2}{m_H^2} + \sin^2(\alpha - \beta) \frac{m_G^2 m_h^2}{m_h^2 - m_G^2} \ln \frac{m_G^2}{m_h^2} \right\}. \end{aligned} \quad (17)$$

Note that the logarithmic terms are all negative. Since  $m_h^2 > m_H^2$ ,  $\delta M_W^2$  is minimized at  $\sin^2(\alpha - \beta) = 1$ .

### 3. Tadpole Diagrams and Self-Energies

Before launching into the computation of the radiative corrections to  $Hw^\dagger w$  vertex, we prepare the counter terms (16) by evaluating  $\delta m_H^2$ ,  $\delta\alpha$ ,  $\delta\beta$ ,  $Z_w$ , and  $Z_H$ . First of all, the condition of the stability of the vacuum (12) must be replaced on the loop-level by including the tadpole diagrams in Fig. 1. The vacuum expectation values  $v_1$  and  $v_2$  are determined in our case by

$$\mu_1^2 = \lambda_1 v_1^2 + \frac{1}{2}(\lambda_3 + \lambda_4)v_2^2 + T_H \frac{\sin \alpha}{v_1} - T_h \frac{\cos \alpha}{v_1}, \quad (18)$$

$$\mu_2^2 = \lambda_2 v_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4)v_1^2 - T_H \frac{\cos \alpha}{v_2} - T_h \frac{\sin \alpha}{v_2}. \quad (19)$$

Here tadpole contributions of Fig. 1 are given by

$$\begin{aligned} T_H = & \frac{3m_H^2}{2v} \left( \frac{\sin^3 \alpha}{\cos \beta} - \frac{\cos^3 \alpha}{\sin \beta} \right) f_1(m_H^2) \\ & - \frac{1}{v} (m_h^2 + \frac{1}{2}m_H^2) \frac{\sin 2\alpha}{\sin 2\beta} \sin(\alpha - \beta) f_1(m_h^2) \\ & + \left\{ \frac{m_H^2}{2v} \left( \frac{\sin \alpha \sin^2 \beta}{\cos \beta} - \frac{\cos \alpha \cos^2 \beta}{\sin \beta} \right) + \frac{m_A^2}{v} \sin(\alpha - \beta) \right\} f_1(m_A^2) \\ & + \left\{ \frac{m_H^2}{v} \left( \frac{\sin \alpha \sin^2 \beta}{\cos \beta} - \frac{\cos \alpha \cos^2 \beta}{\sin \beta} \right) + \frac{2m_G^2}{v} \sin(\alpha - \beta) \right\} f_1(m_G^2), \end{aligned} \quad (20)$$

$$\begin{aligned} T_h = & -\frac{3m_h^2}{2v} \left( \frac{\cos^3 \alpha}{\cos \beta} + \frac{\sin^3 \alpha}{\sin \beta} \right) f_1(m_h^2) \\ & - \frac{1}{v} (m_H^2 + \frac{1}{2}m_h^2) \frac{\sin 2\alpha}{\sin 2\beta} \cos(\alpha - \beta) f_1(m_H^2) \\ & - \left\{ \frac{m_h^2}{2v} \left( \frac{\cos \alpha \sin^2 \beta}{\cos \beta} + \frac{\sin \alpha \cos^2 \beta}{\sin \beta} \right) + \frac{m_A^2}{v} \cos(\alpha - \beta) \right\} f_1(m_A^2) \\ & - \left\{ \frac{m_h^2}{v} \left( \frac{\cos \alpha \sin^2 \beta}{\cos \beta} + \frac{\sin \alpha \cos^2 \beta}{\sin \beta} \right) + \frac{2m_G^2}{v} \cos(\alpha - \beta) \right\} f_1(m_G^2). \end{aligned} \quad (21)$$

The definition of the function  $f_1(m^2)$  is given in Appendix A.

Let us next turn to the self-energy diagrams of  $w$ . The two-point function depicted in Fig. 2 becomes  $2 \times 2$  matrix due to the mixing with charged Higgs boson  $G$ . The two-point functions are decomposed into three terms according to the three diagrams in Figs 2(a), 2(b) and 2(c), respectively,

$$\Pi_{ww}(p^2) = \Pi_{ww}^{(1)} + \Pi_{ww}^{(2)} + \Pi_{ww}^{(3)}, \quad (22)$$

$$\Pi_{wG}(p^2) = \Pi_{wG}^{(1)} + \Pi_{wG}^{(2)} + \Pi_{wG}^{(3)}. \quad (23)$$

Internal particles in Figs. 2(a) and 2(c) are tabulated in Table 1. Our straightforward calculation shows that the sum of the three terms are summarized in the following form

$$\Pi_{ww}(p^2) = \hat{\Pi}_{ww}(p^2) - \hat{\Pi}_{ww}(0), \quad (24)$$

$$\Pi_{wG}(p^2) = \hat{\Pi}_{wG}(p^2) - \hat{\Pi}_{wG}(0). \quad (25)$$

The explicit forms of  $\hat{\Pi}_{ww}(p^2)$  and  $\hat{\Pi}_{wG}(p^2)$  are given in Appendix B. Eqs. (24) and (25) show the existence of a massless pole corresponding to the Nambu-Goldstone boson. For a single Higgs case it has been known that the calculation of the self-energy may be made easier by a manipulation conceived by Taylor [25]. In our case, however, we did not develop similar tricks and just simply added up all Feynman diagrams to reach Eqs. (24) and (25).

The wave function renormalization of the Nambu-Goldstone Boson is finite and turns out to be, for the on-shell renormalization,

$$\begin{aligned} Z_w = 1 - \frac{1}{(4\pi)^2 v^2} \{ & \frac{1}{2}(2m_G^2 + m_H^2 + m_h^2 + m_A^2) + \frac{m_G^2 m_A^2}{m_A^2 - m_G^2} \ln \frac{m_G^2}{m_A^2} \\ & + \cos^2(\alpha - \beta) \frac{m_G^2 m_H^2}{m_H^2 - m_G^2} \ln \frac{m_G^2}{m_H^2} + \sin^2(\alpha - \beta) \frac{m_G^2 m_h^2}{m_h^2 - m_G^2} \ln \frac{m_G^2}{m_h^2} \}. \end{aligned} \quad (26)$$

The renormalization of the mixing angle  $\beta$  is peculiar if we set the renormalization condition at  $p^2 = 0$ . (A different renormalization scheme has been used in Ref. [26].) The vanishing of the off-diagonal part  $\Pi_{wG}(p^2)$  at  $p^2 = 0$  tells us that there is no corrections to the mixing angle  $\beta$ . In other words the relation  $\tan\beta = v_2/v_1$  is preserved to higher orders

$$\delta\beta = -\frac{1}{m_G^2}\Pi_{wG}(0) = 0. \quad (27)$$

The last term in Eq. (16) thus gives no contribution to our calculation.

The other renormalized quantities w.r.t. the  $H$  field are given similarly in terms of the two-point functions,

$$Z_H = 1 + \Pi'_{HH}(m_H^2), \quad (28)$$

$$\delta m_H^2 = \text{Re}(\Pi_{HH}(m_H^2)), \quad (29)$$

$$\delta\alpha = \frac{1}{m_h^2 - m_H^2}\text{Re}(\Pi_{hH}(m_H^2)). \quad (30)$$

It is therefore an urgent task to evaluate  $\Pi_{HH}(p^2)$  and  $\Pi_{hH}(p^2)$ . The relevant Feynman diagrams are again those in Fig. 2 whose internal lines are explained in Table 2. All of the results for these self-energies are given in Appendices C and D.

The mixing phenomena between  $H$  and  $h$  may be understood in the following way. The  $H$  quanta contain the  $h$  component due to the mixing  $\Pi_{hH}(p^2)$ . Therefore  $H$  quanta are able to transform into  $h$  and then couple to  $w^\dagger w$  pair with the strength determined by (14). Such mixing phenomena are already included in (16) through the  $\delta\alpha$  term (times  $m_h^2 \cos(\alpha - \beta)$ ).

## 4. Loop Corrections to the Higgs Vertex

We are now in a position to evaluate the loop effects to the  $Hw^\dagger w$  vertex as shown in Fig. 3. Hereafter the Nambu-Goldstone bosons are put on the mass shell ( $p_1^2 = p_2^2 = 0$ ), but considering future applications to  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$  we will keep the momentum  $p$  carried by  $H$  boson off-shell and will eventually put on the mass-shell ( $p^2 = m_H^2$ ) to get the decay rate.

There are a lot of Feynman diagrams to be calculated. We would like to classify the diagrams into three groups according to the three types in Fig. 4, that is,

$$\Gamma(p^2) = \Gamma^{(1)} + \Gamma^{(2)} + \Gamma^{(3)}. \quad (31)$$

These three terms come from Figs. 4(a), 4(b) and 4(c), respectively. The species of the internal lines in these diagrams are summarized in Table 3.

The sum of all the diagrams in Fig. 4 is rather lengthy, but is given here for our use in numerical calculations. As to the contribution of Fig. 4(a), we obtain

$$\begin{aligned} \Gamma^{(1)} = & -\frac{m_H^6}{v^3} \sin^3(\alpha - \beta) g(p^2, 0, 0, m_H^2) \\ & -\frac{m_h^4 m_H^2}{v^3} \sin(\alpha - \beta) \cos^2(\alpha - \beta) g(p^2, 0, 0, m_h^2) \\ & +\frac{1}{v^3} \left\{ m_H^2 \left( \frac{\cos \alpha \cos^2 \beta}{\sin \beta} - \frac{\sin \alpha \sin^2 \beta}{\cos \beta} \right) - 2m_G^2 \sin(\alpha - \beta) \right\} \\ & \times \{ (m_h^2 - m_G^2)^2 \sin^2(\alpha - \beta) g(p^2, m_G^2, m_G^2, m_h^2) \\ & + (m_H^2 - m_G^2)^2 \cos^2(\alpha - \beta) g(p^2, m_G^2, m_G^2, m_H^2) \\ & + (m_A^2 - m_G^2)^2 g(p^2, m_G^2, m_G^2, m_A^2) \} \\ & +\frac{1}{2v^3} \left\{ m_H^2 \left( \frac{\cos^2 \beta \cos \alpha}{\sin \beta} - \frac{\sin^2 \beta \sin \alpha}{\cos \beta} \right) - 2m_A^2 \sin(\alpha - \beta) \right\} \\ & \times (m_A^2 - m_G^2)^2 g(p^2, m_A^2, m_A^2, m_G^2) \\ & +\frac{m_h^2 (m_H^2 - m_G^2) (m_h^2 - m_G^2)}{v^3} \sin(\alpha - \beta) \cos^2(\alpha - \beta) g(p^2, 0, m_G^2, m_h^2) \\ & -\frac{m_H^2 (m_H^2 - m_G^2)^2}{v^3} \sin(\alpha - \beta) \cos^2(\alpha - \beta) g(p^2, 0, m_G^2, m_H^2) \\ & +\frac{3m_H^6}{v^3} \sin^2(\alpha - \beta) \left( \frac{\cos^3 \alpha}{\sin \beta} - \frac{\sin^3 \alpha}{\cos \beta} \right) g(p^2, m_H^2, m_H^2, 0) \end{aligned}$$

$$\begin{aligned}
& + \frac{3m_H^2(m_H^2 - m_G^2)^2}{v^3} \cos^2(\alpha - \beta) \left( \frac{\cos^3 \alpha}{\sin \beta} - \frac{\sin^3 \alpha}{\cos \beta} \right) g(p^2, m_H^2, m_H^2, m_G^2) \\
& - \frac{m_h^2 m_H^2 (2m_H^2 + m_h^2)}{v^3} \sin(\alpha - \beta) \cos^2(\alpha - \beta) \frac{\sin 2\alpha}{\sin 2\beta} g(p^2, m_H^2, m_h^2, 0) \\
& + \frac{(m_h^2 - m_G^2)(m_H^2 - m_G^2)(2m_H^2 + m_h^2)}{v^3} \sin(\alpha - \beta) \cos^2(\alpha - \beta) \frac{\sin 2\alpha}{\sin 2\beta} \\
& \times g(p^2, m_H^2, m_h^2, m_G^2) \\
& + \frac{m_h^4(m_H^2 + 2m_h^2)}{v^3} \sin(\alpha - \beta) \cos^2(\alpha - \beta) \frac{\sin 2\alpha}{\sin 2\beta} g(p^2, m_h^2, m_h^2, 0) \\
& + \frac{(m_h^2 - m_G^2)^2(m_H^2 + 2m_h^2)}{v^3} \sin^3(\alpha - \beta) \frac{\sin 2\alpha}{\sin 2\beta} g(p^2, m_h^2, m_h^2, m_G^2).
\end{aligned} \tag{32}$$

The Feynman integral is expressed in terms of the function  $g(p^2, m_1^2, m_2^2, m_3^2)$  defined in Appendix A. As is shown there, this function is a combination of the so-called Spence function. (See Ref. [27] for a concise review of the Spence function.) It is almost straightforward to evaluate  $\Gamma^{(1)}$  numerically on computer.

The diagrams in Fig. 4(b) are expressed by the function  $f_2(p^2, m_1^2, m_2^2)$  in defined in Eq. (38) and provide us with

$$\begin{aligned}
\Gamma^{(2)} &= \frac{5m_H^2}{2v^3} \{m_h^2 \cos^2(\alpha - \beta) + m_H^2 \sin^2(\alpha - \beta)\} \sin(\alpha - \beta) f_2(p^2, 0, 0) \\
& - \frac{1}{v^3} \{m_H^2 \left( \frac{\cos \alpha \cos^2 \beta}{\sin \beta} - \frac{\sin \alpha \sin^2 \beta}{\cos \beta} \right) - 2m_G^2 \sin(\alpha - \beta)\} \\
& \times \{2m_h^2 \sin^2(\alpha - \beta) + 2m_H^2 \cos^2(\alpha - \beta) \\
& + \frac{\sin 2\alpha}{\sin 2\beta} (m_h^2 - m_H^2) + m_A^2\} f_2(p^2, m_G^2, m_G^2) \\
& - \frac{1}{v^3} \{m_H^2 \left( \frac{\cos \alpha \cos^2 \beta}{\sin \beta} - \frac{\sin \alpha \sin^2 \beta}{\cos \beta} \right) - 2m_A^2 \sin(\alpha - \beta)\} \\
& \times \left\{ \frac{1}{2} m_h^2 \sin^2(\alpha - \beta) + \frac{1}{2} m_H^2 \cos^2(\alpha - \beta) \right. \\
& + \left. \frac{1}{2} \frac{\sin 2\alpha}{\sin 2\beta} (m_h^2 - m_H^2) + m_G^2 \right\} f_2(p^2, m_A^2, m_A^2) \\
& - \frac{1}{v^3} (m_h^2 - m_H^2)(m_H^2 - m_A^2) \sin(\alpha - \beta) \cos^2(\alpha - \beta) f_2(p^2, m_A^2, 0) \\
& - \frac{4}{v^3} (m_h^2 - m_H^2)(m_H^2 - m_G^2) \sin(\alpha - \beta) \cos^2(\alpha - \beta) f_2(p^2, m_G^2, 0)
\end{aligned}$$

$$\begin{aligned}
& -\frac{3m_H^2}{2v^3}\{((m_h^2 - m_H^2)\frac{\sin 2\alpha}{\sin 2\beta} + 2m_G^2)\cos^2(\alpha - \beta) \\
& + m_H^2\}(\frac{\cos^3 \alpha}{\sin \beta} - \frac{\sin^3 \alpha}{\cos \beta})f_2(p^2, m_H^2, m_H^2) \\
& -\frac{1}{v^3}\{(m_h^2 - m_H^2)\frac{\sin 2\alpha}{\sin 2\beta} + 2m_G^2\}(m_h^2 + 2m_H^2) \\
& \times \sin(\alpha - \beta)\cos^2(\alpha - \beta)\frac{\sin 2\alpha}{\sin 2\beta}f_2(p^2, m_H^2, m_h^2) \\
& -\frac{1}{2v^3}\{((m_h^2 - m_H^2)\frac{\sin 2\alpha}{\sin 2\beta} + 2m_G^2)\sin^2(\alpha - \beta) + m_h^2\} \\
& \times (m_H^2 + 2m_h^2)\sin(\alpha - \beta)\frac{\sin 2\alpha}{\sin 2\beta}f_2(p^2, m_h^2, m_h^2).
\end{aligned} \tag{33}$$

Finally we give the results of Fig. 4(c) which are multiplied by 2, because we have to include symmetric diagrams as well

$$\begin{aligned}
\Gamma^{(3)} = & -\frac{2m_h^2}{v^3}\{(m_h^2 - m_H^2)\frac{\sin 2\alpha}{\sin 2\beta} + 2m_G^2\}\sin(\alpha - \beta)\cos^2(\alpha - \beta)f_2(0, m_h^2, 0) \\
& +\frac{2m_H^2}{v^3}\{(m_h^2\frac{\sin 2\alpha}{\sin 2\beta} + 2m_G^2)\sin(\alpha - \beta)\cos^2(\alpha - \beta) \\
& +m_H^2(\frac{\sin^3 \alpha}{\cos \beta} - \frac{\cos^3 \alpha}{\sin \beta})\sin^2(\alpha - \beta)\}f_2(0, m_H^2, 0) \\
& -\frac{2(m_h^2 - m_G^2)}{v^3}\{\frac{\sin 2\alpha}{\sin 2\beta}(m_h^2\sin^2(\alpha - \beta) + m_H^2\cos^2(\alpha - \beta)) \\
& -m_G^2\cos(2\alpha - 2\beta)\}\sin(\alpha - \beta)f_2(0, m_G^2, m_h^2) \\
& -\frac{m_H^2 - m_G^2}{v^3}\{(m_h^2\frac{\sin 2\alpha}{\sin 2\beta} + 2m_G^2)\sin(2\alpha - 2\beta) \\
& +2m_H^2(\frac{\cos^3 \alpha}{\sin \beta} - \frac{\sin^3 \alpha}{\cos \beta})\cos(\alpha - \beta)\}\cos(\alpha - \beta)f_2(0, m_G^2, m_H^2) \\
& +2\frac{(m_A^2 - m_G^2)^2}{v^3}\sin(\alpha - \beta)f_2(0, m_G^2, m_A^2).
\end{aligned} \tag{34}$$

Putting all the calculations together we arrive at the decay width formula

$$\Gamma(H \rightarrow W_L^+ W_L^-) = \frac{1}{16\pi} \frac{1}{m_H} \sqrt{1 - \frac{4M_W^2}{m_H^2}} |\mathcal{M}(p^2 = m_H^2)|^2. \tag{35}$$

Here the invariant amplitude is given through the one-loop order by the following

sum

$$\begin{aligned}\mathcal{M}(p^2) = & \Gamma(p^2) + \frac{1}{v} \cos(\alpha - \beta) \Pi_{hH}(m_H^2) - \frac{1}{v} \sin(\alpha - \beta) \Pi_{HH}(m_H^2) \\ & + \left\{ \frac{\delta v}{v} + \frac{1}{2} \Pi'_{HH}(m_H^2) + Z_w \right\} \frac{m_H^2}{v} \sin(\alpha - \beta).\end{aligned}\quad (36)$$

Note that, while  $\Gamma(p^2)$ ,  $\Pi_{hH}(m_H^2)$ , and  $\Pi_{HH}(m_H^2)$  are all divergent, the combination of these with the weight in Eq. (36) is finite. Although this is guaranteed by the renormalizability, we have confirmed the finiteness explicitly by hand. This is a non-trivial check of our calculation.

Suppose that  $m_G$  is much larger than all the other masses. It then turns out that  $\Gamma(p^2)$  will be dominated (on the dimensional account) by the terms of the form  $m_G^4/v^3$  with possible logarithmic corrections. The same things also happen to the two-point functions  $\Pi_{HH}(p^2)$  and  $\Pi_{hH}(p^2)$ , which will be principally given a form proportional to  $m_G^4/v^2$ . Thus the decay width is potentially very sensitive to the choice of  $m_G$ , provided that  $m_G$  is large. This situation is in contrast to the standard model, where the effect of the scalar boson is always veiled. Of course the sensitivity to  $m_G$  depends upon the values of  $\alpha$  and  $\beta$ , and it is interesting to see the behavior of the decay width for various choices of  $\alpha$  and  $\beta$ .

## 5. Numerical Analysis of the Decay Width Formula

Now let us analyse the decay width formulae (35) and (36) by putting numbers into the parameters. In principle we should keep an open mind to look at every corner of the parameter space. There have been a lot of efforts to constrain the parameters in the two-Higgs doublet model from phenomenological analyses, and we will give due considerations for an economical purpose to reduce the large parameter space.

Constraints on the mass of the charged Higgs boson ( $m_G$ ) and the mixing angle  $\beta$  has been discussed [28] by considering the low-energy data relating to neutral

meson mixing (  $K^0 - \bar{K}^0$ ,  $D^0 - \bar{D}^0$ ,  $B_d^0 - \bar{B}_d^0$  ) and CP-violation parameters. It has been argued that the low-energy data exclude small  $\tan\beta$  and light  $m_G$  (say,  $\tan\beta < 0.3-0.4$ ,  $m_G < 200$  GeV). In the following we will examine the decay width formula by setting  $\tan\beta = 2$  and 10 as tentative values. These values are within a perturbative region w.r.t. the  $\bar{t}bG$  coupling. We will also vary the charged Higgs boson mass as  $400 \text{ GeV} < m_G < 900 \text{ GeV}$ . For the values in this region it has been known [6] that the  $\rho$  parameter and the radiative corrections to the muon decay constant do not contradict the present experimental values. Since the  $H$  boson is assumed to be the lightest, we fix  $m_H = 300 \text{ GeV}$  and also we take  $m_h = 400 \text{ GeV}$ . These values satisfy the tree unitarity constraint as mentioned in Introduction.

As to the choice of the angle  $\alpha$ , there does not seem to be a thorough phenomenological analysis comparable to the case of  $\beta$ . Alternatively, the angle  $\alpha$  may be fixed in a theoretically oriented way. In the minimal supersymmetric model for example, the parameter  $\alpha$  is not independent but expressed in terms of the Higgs boson and gauge boson masses. If one takes the charged Higgs boson mass as an independent parameter and assumes an extremely large value for it, then  $\sin^2(\alpha - \beta)$  approaches to unity. In this case the strength of the  $Hw^\dagger w$  vertex (13) becomes maximal. In the following we will examine two extreme cases, namely,  $\sin^2(\alpha - \beta) = 1$  and  $\sin^2(\alpha - \beta) = 0$ . Note that, in the latter case, the  $Hw^\dagger w$  vertex vanishes on the tree level, and the decay process proceeds only due to loop effects.

To sum up we will evaluate the decay width (35) as a function of  $m_G$  for the following cases.

Case 1:  $\tan\alpha = \tan\beta = 2$ ,  $m_H = 300 \text{ GeV}$ ,  $m_h = 400 \text{ GeV}$ ,

Case 2:  $\tan\alpha = \tan\beta = 10$ ,  $m_H = 300 \text{ GeV}$ ,  $m_h = 400 \text{ GeV}$ ,

Case 3:  $\tan\beta = 2$ ,  $\sin^2(\alpha - \beta) = 1$ ,  $m_H = 300 \text{ GeV}$ ,  $m_h = 400 \text{ GeV}$ ,

Case 4:  $\tan\beta = 10$ ,  $\sin^2(\alpha - \beta) = 1$ ,  $m_H = 300 \text{ GeV}$ ,  $m_h = 400 \text{ GeV}$ ,

As to the mass  $m_A$ , we will set tentatively  $m_A = 350$  GeV,  $m_A = 700$  GeV, and  $m_A = 1000$  GeV. The result of our numerical computation for the cases 1, 2, 3 and 4 are illustrated in Figs. 5, 6, 7 and 8, respectively.

For the sake of comparison let us recall the decay width formula in the one Higgs boson case. The decay width in the minimal standard model (MSM) is given through the one-loop order by [12]

$$\begin{aligned} \Gamma^{\text{MSM}}(H \rightarrow W_L^+ W_L^-) &= \frac{1}{16\pi} \frac{m_H^3}{v^2} \sqrt{1 - \frac{4M_W^2}{m_H^2}} \\ &\times |1 + \frac{1}{4\pi^2} \frac{m_H^2}{v^2} (\frac{19}{16} - \frac{3\sqrt{3}\pi}{8} + \frac{5\pi^2}{48})|^2. \end{aligned} \quad (37)$$

This formula gives us  $\Gamma^{\text{MSM}}(H \rightarrow W_L^+ W_L^-) \sim 7.6$  GeV for  $m_H = 300$  GeV. Note that the radiative correction in (37) is as small as 1.3 %. This is due to the smallness of the coefficients of  $m_H^2/v^2$  in (37).

Let us now look at Figs. 5,6,7 and 8 closely. We can immediately see conspicuous differences in the magnitude of computed values in these four cases. The cases 3 and 4 (Figs. 7 and 8) show that the computed width ranges from 1 to 8 GeV, depending on  $m_G$  and  $m_A$ . In the case 1, on the other hand, the width is smaller by one or two orders of magnitude or perhaps more depending on  $m_G$ , compared to the cases 3 and 4. The case 2 also gives small decay width for  $400 \text{ GeV} < m_G < 700 \text{ GeV}$ .

This situation may be understood in the following way. As mentioned before, in the cases 1 and 2, the tree level coupling vanishes and the predicted decay width is only due to the radiative corrections. The radiative corrections thus turn out to be small for the cases 1 and 2 for the same reason as in the MSM case. This is particularly interesting, if one would notice that the values  $m_G = 900$  GeV and  $m_A = 1000$  GeV are on the verge of the breakdown of perturbative calculation.

We also note that the  $m_G$ -dependence of the width for cases 1 and 2 is wild-behaved. It changes by one or two orders of magnitude as we vary  $m_G$  from 400 GeV to 900 GeV. For large- $m_G$ , the width goes up like  $m_G^8$ , which is in accordance

with our initial expectation. At any rate, if the width would be this small as in Fig. 5, the experimental measurement of the width would not be easy.

Since we have set  $\sin^2(\alpha - \beta) = 1$  for the cases 3 and 4, the tree-level prediction of the width for these cases coincide with the tree-level value in the MSM case without radiative correction, which becomes 7.5 GeV. In Figs. 7 and 8, we see that the radiative correction becomes important for large  $m_G$  for  $m_A = 350$  GeV and  $m_A = 700$  GeV. The corrections reaches 35 % (18 %) for  $m_A = 350$  GeV ( $m_A = 700$  GeV) at  $m_G = 900$  GeV. If the  $m_A$  is as large as 1000 GeV, the radiative corrections become comparable with the tree-level calculation for small  $m_G$ . This is attributed to the largeness of  $m_A$  and shows that the perturbative calculation is barely allowed for this choice of  $m_A$ . The  $m_G$ -dependence in Figs. 7 and 8 is not so wild as in Figs. 5 and 6. This is due to the fact that the wild  $m_G^4/v^3$  behavior of the loop amplitudes is tamed by larger  $m_G$ -independent tree amplitudes.

We have repeated the above calculation by setting  $m_h = 600$  GeV. The results, however, do not differ much from those shown in Figs. 5,6,7 and 8. This fact is also very interesting, since the value  $m_h = 600$  GeV is close to the boundary of the perturbative region. This insensitivity to  $m_h$  reminds us of the analysis by Bertolini [6]. He studied radiative corrections, in the two-doublet model, to the  $\rho$ -parameter and to the relation between the Fermi constant and the  $SU(2)_L$  gauge coupling (often denoted by  $\Delta r$ ). He argued that, for  $\alpha = \beta$ ,  $\Delta r$  is independent of  $m_h$ .

Suppose that the Higgs boson  $H$  has been discovered and the decay width  $\Gamma(H \rightarrow W_L^+ W_L^-)$  has been measured to a good accuracy. In such a situation, the calculation shown in Figs. 7 and 8 may be useful to probe the existence of another Higgs boson, *i.e.* G and A if we know the values of  $\alpha$  and  $\beta$ . Of course we do not know at present definite value of  $\alpha$  or  $\beta$ , either. We have to look for various means of measuring the parameters of the two-doublet model. Being given various measurements, we will be able to fit the data in a multi-dimensional parameter space.

Finally we add a comment of the  $p^2$  dependence of the invariant amplitude  $\mathcal{M}(p^2)$ . We have examined  $\mathcal{M}(p^2)$  numerically in the hope of possibility of future confrontation with measurement. With the parameters of cases 3 and 4, this form-factor is almost a constant for  $200 \text{ GeV} < \sqrt{p^2} < 350 \text{ GeV}$ ,  $m_G^2 = 500 \text{ GeV}$ ,  $m_A^2 = 350 \text{ GeV}$ ,  $700 \text{ GeV}$ , and  $1000 \text{ GeV}$ , and does not show a peculiar behavior. This is because the  $p^2$ -independent tree amplitude dominates over the loop amplitudes. In the cases 1 and 2, on the other hand, where the tree amplitude vanishes, the form factor goes up rapidly as we increase  $\sqrt{p^2}$  from  $250 \text{ GeV}$  to  $350 \text{ GeV}$ . Its magnitude is, however, still very small compared to the cases 3 and 4.

## 6. Summary and Discussions

In the present paper, we have investigated the radiative corrections in the two-Higgs doublet model from the viewpoint that the loop corrections to the decay rate of  $H \rightarrow W_L^+ W_L^-$  could be a useful probe into the other Higgs bosons. The corrections to the  $H w^\dagger w$  vertex contain terms behaving like  $m_G^4/v^3$  for large  $m_G$ . This indicates that predicted width could be sensitive to the value  $m_G$ . We have demonstrated the behavior of the width as a function of  $m_G$  for various choices of  $m_h$ ,  $m_A$ ,  $\alpha$  and  $\beta$ . We have seen to what extent the terms of the power-behavior would become important in the decay width formula.

The calculation presented in this paper can be easily carried over to the decay  $H \rightarrow Z_L Z_L$ , the analysis of which will be given in our future publication [29]. The elastic scattering  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ ,  $Z_L Z_L$  is also of particular interest. The present calculation will become a useful basis for the evaluation of these elastic scattering.

The Higgs boson masses and the radiative corrections in supersymmetric theories have been one of important topics in recent literatures [30]. The parameters in the Higgs sector in supersymmetric theories are not completely independent, but are

constrained by the supersymmetry. Suppose that the decay rate of  $H \rightarrow W_L^+ W_L^-$  be evaluated in supersymmetric models. The parameters describing the decay rate can not be varied freely. For such a case, it might be that the screening phenomena could occur effectively, because of the constrained parameter space. We will come to this problem in the future.

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## Appendix A

Here we summarize our notations for the Feynman integrals corresponding to various types of diagrams in Fig. 2, which are expressed by

$$\begin{aligned}
f_1(m^2) &= \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2 - m^2} \\
&= \frac{m^2}{(4\pi)^2} \left( \frac{2}{D-4} - 1 + \gamma_E + \ln \frac{m^2}{4\pi\mu^2} \right), \\
f_2(p^2, m_1^2, m_2^2) &= -i\mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2 - m_1^2} \frac{i}{(p-k)^2 - m_2^2}.
\end{aligned} \tag{38}$$

Here  $\mu$  is the scale parameter of the  $D$ -dimensional regularization. These notations will be used extensively in Appendices B, C, and D. If we set  $m_2^2 = 0$  in Eq. (38), we obtain

$$\begin{aligned}
f_2(p^2, m^2, 0) &= \frac{1}{(4\pi)^2} \left\{ \frac{2}{D-4} + \gamma_E - 2 + \ln \frac{m^2}{4\pi\mu^2} \right. \\
&\quad \left. + \left(1 - \frac{m^2}{p^2}\right) \ln \left(1 - \frac{p^2}{m^2}\right) \right\}.
\end{aligned} \tag{39}$$

The formula (38) is also simplified if we put  $m_1 = m_2 = m$ , i.e.,

$$\begin{aligned}
f_2(p^2, m^2, m^2) &= \frac{1}{(4\pi)^2} \left\{ \frac{2}{D-4} + \gamma_E - 2 + \ln \frac{m^2}{4\pi\mu^2} \right. \\
&\quad \left. + \sqrt{1 - \frac{4m^2}{p^2}} \ln \frac{\sqrt{1 - 4m^2/p^2} + 1}{\sqrt{1 - 4m^2/p^2} - 1} \right\}.
\end{aligned} \tag{40}$$

The vertex integral corresponding to Fig. 4 is expressed in terms of the following function

$$g(p^2, m_1^2, m_2^2, m_3^2) = \int \frac{d^4 k}{(2\pi)^4} \frac{i}{(k-p_1)^2 - m_1^2} \frac{i}{(k+p_2)^2 - m_2^2} \frac{i}{k^2 - m_3^2}. \tag{41}$$

It has been known that this integral is given by a combination of the Spence function;

$$\begin{aligned}
g(p^2, m_1^2, m_2^2, m_3^2) &= \frac{1}{(4\pi)^2 p^2} \ln\left(\frac{a-1}{a}\right) \{\ln(a - \xi_+) + \ln(a - \xi_-)\} \\
&+ \ln\left(\frac{p^2}{(a+b)(m_1^2 - m_3^2)}\right) \} \\
&+ \frac{1}{(4\pi)^2 p^2} \{-\text{Sp}\left(\frac{a-1}{a - \xi_+}\right) + \text{Sp}\left(\frac{a}{a - \xi_+}\right) - \text{Sp}\left(\frac{a-1}{a - \xi_-}\right) \\
&+ \text{Sp}\left(\frac{a}{a - \xi_-}\right) + \text{Sp}\left(\frac{a-1}{a+b}\right) - \text{Sp}\left(\frac{a}{a+b}\right)\}. \tag{42}
\end{aligned}$$

Here the Spence function is defined as usual by

$$\text{Sp}(z) = - \int_0^z \frac{dx}{x} \ln(1-x). \tag{43}$$

Various quantities appearing in (42) are given by

$$\xi_{\pm} = \frac{m_2^2 - m_1^2 + p^2 \pm \sqrt{(m_2^2 - m_1^2 + p^2)^2 - 4m_2^2 p^2}}{2p^2}, \tag{44}$$

$$a = \frac{m_2^2 - m_3^2}{p^2}, \quad b = \frac{m_3^2}{m_1^2 - m_3^2}. \tag{45}$$

## Appendix B

The self-energy of the Nambu-Goldstone boson  $\Pi_{ww}(p^2)$ , and the two-point function  $\Pi_{wG}(p^2)$  are obtained by evaluating Fig. 2, whose internal particles are specified in Table 1. Straightforward calculations show that these functions are expressed as in (24) and (25), where we have introduced following functions

$$\begin{aligned}
\hat{\Pi}_{ww}(p^2) &= \frac{m_h^4}{v^2} \cos^2(\alpha - \beta) f_2(p^2, m_h^2, 0) \\
&+ \frac{m_H^4}{v^2} \sin^2(\alpha - \beta) f_2(p^2, m_H^2, 0) \\
&+ \frac{(m_h^2 - m_G^2)^2}{v^2} \sin^2(\alpha - \beta) f_2(p^2, m_h^2, m_G^2) \\
&+ \frac{(m_H^2 - m_G^2)^2}{v^2} \cos^2(\alpha - \beta) f_2(p^2, m_H^2, m_G^2) \\
&+ \frac{(m_A^2 - m_G^2)^2}{v^2} f_2(p^2, m_A^2, m_G^2), \tag{46}
\end{aligned}$$

$$\begin{aligned}
\hat{\Pi}_{wG}(p^2) = & \frac{m_h^2(m_h^2 - m_G^2)}{2v^2} \sin(2\alpha - 2\beta) f_2(p^2, m_h^2, 0) \\
& - \frac{m_H^2(m_H^2 - m_G^2)}{2v^2} \sin(2\alpha - 2\beta) f_2(p^2, m_H^2, 0) \\
& + \frac{m_h^2 - m_G^2}{v^2} \sin(\alpha - \beta) \left\{ m_h^2 \left( \frac{\sin^2 \beta \cos \alpha}{\cos \beta} + \frac{\cos^2 \beta \sin \alpha}{\sin \beta} \right) \right. \\
& + 2m_G^2 \cos(\alpha - \beta) \left. \right\} f_2(p^2, m_h^2, m_G^2) \\
& + \frac{m_H^2 - m_G^2}{v^2} \cos(\alpha - \beta) \left\{ m_H^2 \left( \frac{\cos^2 \beta \cos \alpha}{\sin \beta} - \frac{\sin^2 \beta \sin \alpha}{\cos \beta} \right) \right. \\
& - 2m_G^2 \sin(\alpha - \beta) \left. \right\} f_2(p^2, m_H^2, m_G^2).
\end{aligned} \tag{47}$$

### Appendix C

The self-energy of the  $H$  boson is decomposed into three parts

$$\Pi_{HH}(p^2) = \Pi_{HH}^{(1)} + \Pi_{HH}^{(2)} + \Pi_{HH}^{(3)}, \tag{48}$$

which correspond to Figs. 2(a), 2(b) and 2(c), respectively. These three diagrams give us the following results:

$$\begin{aligned}
\Pi_{HH}^{(1)} = & \frac{9m_H^4}{2v^2} \left( \frac{\cos^3 \alpha}{\sin \beta} - \frac{\sin^3 \alpha}{\cos \beta} \right)^2 f_2(p^2, m_H^2, m_H^2) \\
& + \frac{(2m_H^2 + m_h^2)^2}{v^2} \left( \frac{\sin 2\alpha}{\sin 2\beta} \right)^2 \cos^2(\alpha - \beta) f_2(p^2, m_h^2, m_H^2) \\
& + \frac{(2m_h^2 + m_H^2)^2}{2v^2} \left( \frac{\sin 2\alpha}{\sin 2\beta} \right)^2 \sin^2(\alpha - \beta) f_2(p^2, m_h^2, m_h^2) \\
& + \frac{3m_H^4}{2v^2} \sin^2(\alpha - \beta) f_2(p^2, 0, 0) \\
& + \left[ \frac{m_H^2}{v} \left( \frac{\cos^2 \beta \cos \alpha}{\sin \beta} - \frac{\sin^2 \beta \sin \alpha}{\cos \beta} \right) - \frac{2m_G^2}{v} \sin(\alpha - \beta) \right]^2 \\
& \times f_2(p^2, m_G^2, m_G^2) \\
& + \frac{1}{2} \left[ \frac{m_H^2}{v} \left( \frac{\cos^2 \beta \cos \alpha}{\sin \beta} - \frac{\sin^2 \beta \sin \alpha}{\cos \beta} \right) - \frac{2m_A^2}{v} \sin(\alpha - \beta) \right]^2 \\
& \times f_2(p^2, m_A^2, m_A^2)
\end{aligned}$$

$$\begin{aligned}
& + \frac{(m_H^2 - m_A^2)^2}{v^2} \cos^2(\alpha - \beta) f_2(p^2, m_A^2, 0) \\
& + \frac{2(m_H^2 - m_G^2)^2}{v^2} \cos^2(\alpha - \beta) f_2(p^2, m_G^2, 0),
\end{aligned} \tag{49}$$

$$\begin{aligned}
\Pi_{HH}^{(2)} = & \left[ \frac{m_h^2}{2v^2} \left( \frac{\cos \alpha \sin \beta}{\cos^2 \beta} + \frac{\sin \alpha \cos \beta}{\sin^2 \beta} \right) \sin 2\alpha \cos(\alpha - \beta) \right. \\
& + \frac{m_H^2}{v^2} \left( \frac{\cos^3 \alpha}{\sin \beta} - \frac{\sin^3 \alpha}{\cos \beta} \right) \left( \frac{\cos \alpha \cos^2 \beta}{\sin \beta} - \frac{\sin \alpha \sin^2 \beta}{\cos \beta} \right) \\
& + \frac{2m_G^2}{v^2} \sin^2(\alpha - \beta) \left. \right] f_1(m_G^2) \\
& + \left[ \frac{m_h^2}{4v^2} \left( \frac{\cos \alpha \sin \beta}{\cos^2 \beta} + \frac{\sin \alpha \cos \beta}{\sin^2 \beta} \right) \sin 2\alpha \cos(\alpha - \beta) \right. \\
& + \frac{m_H^2}{2v^2} \left( \frac{\cos^3 \alpha}{\sin \beta} - \frac{\sin^3 \alpha}{\cos \beta} \right) \left( \frac{\cos \alpha \cos^2 \beta}{\sin \beta} - \frac{\sin \alpha \sin^2 \beta}{\cos \beta} \right) \\
& + \frac{m_A^2}{v^2} \sin^2(\alpha - \beta) \left. \right] f_1(m_A^2) \\
& + \left[ \frac{m_h^2}{2v^2} \left\{ 3 \left( \frac{\sin 2\alpha}{\sin 2\beta} \right)^2 \sin^2(\alpha - \beta) + \frac{\sin 2\alpha}{\sin 2\beta} \right\} \right. \\
& + \frac{m_H^2}{2v^2} \left\{ 3 \left( \frac{\sin 2\alpha}{\sin 2\beta} \right)^2 \cos^2(\alpha - \beta) - \frac{\sin 2\alpha}{\sin 2\beta} \right\} \left. \right] f_1(m_h^2) \\
& + \left[ \frac{3m_h^2}{2v^2} \left( \frac{\sin^2 \alpha \cos \alpha}{\cos \beta} + \frac{\sin \alpha \cos^2 \alpha}{\sin \beta} \right)^2 + \frac{3m_H^2}{2v^2} \left( \frac{\sin^3 \alpha}{\cos \beta} - \frac{\cos^3 \alpha}{\sin \beta} \right)^2 \right] \\
& \times f_1(m_H^2),
\end{aligned} \tag{50}$$

$$\Pi_{HH}^{(3)} = \frac{1}{v} \left( \frac{\cos^3 \alpha}{\sin \beta} - \frac{\sin^3 \alpha}{\cos \beta} \right) T_H + \frac{1}{v} \frac{\sin 2\alpha}{\sin 2\beta} \cos(\alpha - \beta) T_h. \tag{51}$$

## Appendix D

The mixing diagrams between  $H$  and  $h$  are also of the types of Fig. 2. They are again decomposed into three terms

$$\Pi_{hH}(p^2) = \Pi_{hH}^{(1)} + \Pi_{hH}^{(2)} + \Pi_{hH}^{(3)}, \tag{52}$$

where each term in (52) are given by

$$\begin{aligned}
\Pi_{hH}^{(1)} = & \frac{1}{2v^2}(2m_h^2 + m_H^2)(2m_H^2 + m_h^2)\left(\frac{\sin 2\alpha}{\sin 2\beta}\right)^2 \sin(2\alpha - 2\beta) f_2(p^2, m_h^2, m_H^2) \\
& + \frac{3m_h^2}{v^2}\left(m_h^2 + \frac{1}{2}m_H^2\right)\left(\frac{\cos^3 \alpha}{\cos \beta} + \frac{\sin^3 \alpha}{\sin \beta}\right) \frac{\sin 2\alpha}{\sin 2\beta} \sin(\alpha - \beta) \\
& \times f_2(p^2, m_h^2, m_h^2) \\
& + \frac{3m_H^2}{v^2}\left(m_H^2 + \frac{1}{2}m_h^2\right)\left(\frac{\cos^3 \alpha}{\sin \beta} - \frac{\sin^3 \alpha}{\cos \beta}\right) \frac{\sin 2\alpha}{\sin 2\beta} \cos(\alpha - \beta) \\
& \times f_2(p^2, m_H^2, m_H^2) \\
& - \frac{3m_h^2 m_H^2}{4v^2} \sin(2\alpha - 2\beta) f_2(p^2, 0, 0) \\
& + \left\{ \frac{m_h^2}{v} \left( \frac{\sin^2 \beta \cos \alpha}{\cos \beta} + \frac{\sin \alpha \cos^2 \beta}{\sin \beta} \right) + \frac{2m_G^2}{v} \cos(\alpha - \beta) \right\} \\
& \times \left\{ \frac{m_H^2}{v} \left( \frac{\cos^2 \beta \cos \alpha}{\sin \beta} - \frac{\sin \alpha \sin^2 \beta}{\cos \beta} \right) - \frac{2m_G^2}{v} \sin(\alpha - \beta) \right\} \\
& \times f_2(p^2, m_G^2, m_G^2) \\
& + \frac{1}{2} \left\{ \frac{m_h^2}{v} \left( \frac{\sin^2 \beta \cos \alpha}{\cos \beta} + \frac{\sin \alpha \cos^2 \beta}{\sin \beta} \right) + \frac{2m_A^2}{v} \cos(\alpha - \beta) \right\} \\
& \times \left\{ \frac{m_H^2}{v} \left( \frac{\cos^2 \beta \cos \alpha}{\sin \beta} - \frac{\sin \alpha \sin^2 \beta}{\cos \beta} \right) - \frac{2m_A^2}{v} \sin(\alpha - \beta) \right\} \\
& \times f_2(p^2, m_A^2, m_A^2) \\
& + \frac{1}{v^2} (m_h^2 - m_G^2)(m_H^2 - m_G^2) \sin(2\alpha - 2\beta) f_2(p^2, m_G^2, 0) \\
& + \frac{1}{2v^2} (m_h^2 - m_A^2)(m_H^2 - m_A^2) \sin(2\alpha - 2\beta) f_2(p^2, m_A^2, 0),
\end{aligned} \tag{53}$$

$$\begin{aligned}
\Pi_{hH}^{(2)} = & \left\{ \frac{m_h^2}{2v^2} \left( \frac{\cos \alpha \sin \beta}{\cos^2 \beta} + \frac{\sin \alpha \cos \beta}{\sin^2 \beta} \right) \sin 2\alpha \sin(\alpha - \beta) \right. \\
& + \frac{m_H^2}{2v^2} \left( \frac{\cos \alpha \cos \beta}{\sin^2 \beta} - \frac{\sin \alpha \sin \beta}{\cos^2 \beta} \right) \sin 2\alpha \cos(\alpha - \beta) \\
& - \frac{m_G^2}{v^2} \sin(2\alpha - 2\beta) \left. \right\} f_1(m_G^2) \\
& + \left\{ \frac{m_h^2}{4v^2} \left( \frac{\cos \alpha \sin \beta}{\cos^2 \beta} + \frac{\sin \alpha \cos \beta}{\sin^2 \beta} \right) \sin 2\alpha \sin(\alpha - \beta) \right. \\
& + \frac{m_H^2}{4v^2} \left( \frac{\cos \alpha \cos \beta}{\sin^2 \beta} - \frac{\sin \alpha \sin \beta}{\cos^2 \beta} \right) \sin 2\alpha \cos(\alpha - \beta)
\end{aligned}$$

$$\begin{aligned}
& -\frac{m_A^2}{2v^2} \sin(2\alpha - 2\beta) \} f_1(m_A^2) \\
& + \{ \frac{3m_h^2}{4v^2} (\frac{\sin 2\alpha}{\sin 2\beta})^2 \sin 2(\alpha - \beta) \\
& + \frac{3m_H^2}{2v^2} \frac{\sin 2\alpha}{\sin 2\beta} (\frac{\cos^3 \alpha}{\sin \beta} - \frac{\sin^3 \alpha}{\cos \beta}) \cos(\alpha - \beta) \} f_1(m_H^2) \\
& + \{ \frac{3m_H^2}{4v^2} (\frac{\sin 2\alpha}{\sin 2\beta})^2 \sin 2(\alpha - \beta) \\
& + \frac{3m_h^2}{2v^2} \frac{\sin 2\alpha}{\sin 2\beta} (\frac{\cos^3 \alpha}{\cos \beta} + \frac{\sin^3 \alpha}{\sin \beta}) \sin(\alpha - \beta) \} f_1(m_h^2),
\end{aligned} \tag{54}$$

$$\Pi_{hH}^{(3)} = \frac{1}{v} \frac{\sin 2\alpha}{\sin 2\beta} \{ \cos(\alpha - \beta) T_H + \sin(\alpha - \beta) T_h \}. \tag{55}$$

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**Table 1**

Combinations of internal particles  $(X, Y)$  running in Fig. 2(a) and  $X$  in Fig. 2(b) for  $\Pi_{ww}^{(i)}$  and  $\Pi_{wG}^{(i)}$  ( $i = 1, 2$ ).

| Propagator       | Internal particle species                |
|------------------|--|
| $\Pi_{ww}^{(1)}$ | $(h, w), (H, w), (h, G), (H, G), (A, G)$ |
| $\Pi_{ww}^{(2)}$ | $G, A, h, H$                             |
| $\Pi_{wG}^{(1)}$ | $(h, w), (H, w), (h, G), (H, G)$         |
| $\Pi_{wG}^{(2)}$ | $G, A, h, H$                             |

**Table 2**

Combinations of internal particles  $(X, Y)$  running in Fig. 2(a) and  $X$  in Fig. 2(b) for  $\Pi_{HH}^{(i)}$  and  $\Pi_{hH}^{(i)}$  ( $i = 1, 2$ ).

| Propagator                       | Internal particle species   |
|----------------------------------|---|
| $\Pi_{HH}^{(1)}, \Pi_{hH}^{(1)}$ | $(H, H), (h, H), (h, h), (w, w), (z, z),$<br>$(G, G), (A, A), (A, z), (G, w)$ |
| $\Pi_{HH}^{(2)}, \Pi_{hH}^{(2)}$ | $G, A, h, H$  |

**Table 3**

Combinations of internal particles  $(X, Y, ; Z)$  in Fig. 4(a) and  $(X, Y)$  in Figs. 4(b) and 4(c) for the vertices  $\Gamma^{(i)}$  ( $i = 1, 2, 3$ ).

| Vertex         | Internal particle species  |
|----------------|--|
| $\Gamma^{(1)}$ | $(w, w; h), (w, w; H), (G, G; h), (G, G; H),$<br>$(G, G; A), (A, A; G), (G, w; h), (w, G; h),$<br>$(G, w; H), (w, G; H), (H, H; w), (H, H; G),$<br>$(H, h; w), (h, H; w), (H, h; G), (h, H; G),$<br>$(h, h; w), (h, h; G)$ |
| $\Gamma^{(2)}$ | $(w, w), (z, z), (G, G), (A, A), (A, z),$<br>$(G, w), (H, H), (H, h), (h, h)$  |
| $\Gamma^{(3)}$ | $(w, h), (w, H), (G, h), (G, H), (G, A)$   |

## Figure Captions

Fig. 1

Tadpole diagrams of  $H$  and  $h$  fields contributing to (a)  $T_H$  and (b)  $T_h$ .

Fig. 2

Self-Energy diagrams contributing to (a)  $\Pi_{ij}^{(1)}$ , (b)  $\Pi_{ij}^{(2)}$ , and (c)  $\Pi_{ij}^{(3)}$ , respectively. The diagram (c) denotes the counter terms, which are expressed by the tadpole contributions. A pair of indices  $(i, j)$  refers to either of  $(w, w)$ ,  $(w, G)$ ,  $(H, H)$ , or  $(h, H)$ . Internal particle species are given in Tables 1 and 2.

Fig. 3

General configuration of the  $Hww^\dagger$  vertex. The Nambu-Goldstone bosons are put on the mass shell ( $p_1^2 = p_2^2 = 0$ ), while the Higgs boson  $H$  are kept off-shell to keep generality.

Fig. 4

Radiative corrections to the  $Hw^\dagger w$  vertex contributing to (a)  $\Gamma^{(1)}$ , (b)  $\Gamma^{(2)}$  and (c)  $\Gamma^{(3)}$ . Internal particle species  $(X, Y; Z)$  in (a) and  $(X, Y)$  in (b) and those in (c) are given in Table 3.

Fig. 5

The decay width (35) as a function of  $m_G$ . The mixing angles are determined by  $\tan \alpha = \tan \beta = 2$ . The masses of the neutral Higgs bosons are assumed to be  $m_H = 300$  GeV,  $m_h = 400$  GeV. The CP-odd Higgs boson mass is taken as (a)  $m_A = 350$  GeV (solid line), (b)  $m_A = 700$  GeV (dashed line) and (c)  $m_A = 1000$  GeV (dotted line), respectively.

Fig. 6

The decay width (35) as a function of  $m_G$ . The mixing angles are determined by  $\tan \alpha = \tan \beta = 10$ . The masses of the neutral Higgs bosons are assumed to be  $m_H = 300$  GeV,  $m_h = 400$  GeV. The CP-odd Higgs boson mass is taken as (a)  $m_A = 350$  GeV (solid line), (b)  $m_A = 700$  GeV (dashed line) and (c)  $m_A = 1000$  GeV (dotted line), respectively.

Fig. 7

The decay width (35) as a function of  $m_G$ . The mixing angles are determined by  $\tan \beta = 2$  and  $\sin^2(\alpha - \beta) = 1$ . The masses of the neutral Higgs bosons are assumed to be  $m_H = 300$  GeV,  $m_h = 400$  GeV. The CP-odd Higgs boson mass is taken as (a)  $m_A = 350$  GeV (solid line), (b)  $m_A = 700$  GeV (dashed line) and (c)  $m_A = 1000$  GeV (dotted line), respectively.

Fig. 8

The decay width (35) as a function of  $m_G$ . The mixing angles are determined by  $\tan \beta = 10$  and  $\sin^2(\alpha - \beta) = 1$ . The masses of the neutral Higgs bosons are assumed to be  $m_H = 300$  GeV,  $m_h = 400$  GeV. The CP- odd Higgs boson mass is taken as (a)  $m_A = 350$  GeV (solid line), (b)  $m_A = 700$  GeV (dashed line) and (c)  $m_A = 1000$  GeV (dotted line), respectively.

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